

Contributions of vacuum and plasmon modes to the force on a small sphere near a plate

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2006 J. Phys. A: Math. Gen. 39 6365

(<http://iopscience.iop.org/0305-4470/39/21/S29>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.104

The article was downloaded on 03/06/2010 at 04:30

Please note that [terms and conditions apply](#).

Contributions of vacuum and plasmon modes to the force on a small sphere near a plate

L H Ford

Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, MA 02155, USA

E-mail: ford@cosmos.phy.tufts.edu

Received 14 November 2005, in final form 6 January 2006

Published 10 May 2006

Online at stacks.iop.org/JPhysA/39/6365

Abstract

The force on a small sphere with a plasma model dielectric function and in the presence of a perfectly reflecting plane is considered. The contribution of both the vacuum modes of the quantized electromagnetic field and of plasmon modes in the sphere are discussed. In the case that the plasmon modes are in their ground state, quasi-oscillatory terms from the vacuum and plasmon parts cancel one another, leading a monotonic attractive force. If the plasmon modes are not in the ground state, the net force is quasi-oscillatory. In both cases, the sphere behaves in the same way as does an atom in either its ground state or an excited state.

PACS number: 12.20.–m

1. Introduction

As is well known, Casimir and Polder [1] calculated the interaction energy between an atom in its ground state and a perfectly reflecting plate and obtained the result

$$V = -\frac{1}{4\pi z^3} \int_0^\infty d\xi \alpha(i\xi)(2z^2\xi^2 + 2z\xi + 1) e^{-2z\xi}, \quad (1)$$

where z is the distance between the atom and the plate and $\alpha(i\xi)$ is the atomic polarizability evaluated at imaginary frequency. In the limit of large z , this potential takes the simple form

$$V \sim V_a = -\frac{3\alpha(0)}{8\pi z^4}, \quad (2)$$

which can be interpreted as due to the interaction of the atom with the quantized electromagnetic field. In particular, the asymptotic Casimir–Polder potential can be expressed as

$$V_a = -\frac{1}{2}\alpha(0)\langle \mathbf{E}^2 \rangle = \frac{\alpha(0)}{4\pi z^3} \int_0^\infty d\omega [(2\omega^2 z^2 - 1) \sin 2\omega z + 2\omega z \cos 2\omega z]. \quad (3)$$

Here, $\langle \mathbf{E}^2 \rangle$ is the shift in the mean square of the quantized electric field due to the plate, and the integral on ω is understood to be evaluated by use of a convergence factor. For example, insert a factor of $e^{-\beta\omega}$ in the integrand and then take the limit $\beta \rightarrow 0$ after evaluation of the integral. The integrand is a function which is highly oscillatory, and the area under each oscillation is much larger than the final result for the integral. Thus, if one could slightly modify the frequency spectrum, which is the integrand in equation (3), both the magnitude and sign of the result could be dramatically altered. The Casimir effect between parallel plates also exhibits a quasi-oscillatory spectrum [2, 3], and the possible effects of its modification were discussed in [4].

In a previous paper, [5], the force on a small dielectric sphere in the presence of a perfectly reflecting plate was discussed. It was shown that the interaction of the sphere with the quantized electromagnetic field produces a quasi-oscillatory force whose magnitude can be much larger than the force associated with the asymptotic Casimir–Polder potential, equation (2). This result suggests a modification of the frequency spectrum of vacuum fluctuations. More recently, the model in [5] was extended to include finite reflectivity of the wall and finite temperature effects [6]. However, Barton [7] has suggested that the results in [5, 6] are incomplete in that they do not include a contribution from the quantum charge fluctuations in the sphere, and that this effect will cancel the quasi-oscillatory terms when the sphere is in its ground state. This contribution, also known as radiation reaction or the effect of plasmonic modes, arises from the retarded self-interaction of quantum charge fluctuations in the presence of the mirror.

As will be discussed below, Barton’s suggestion is correct; the quasi-oscillatory terms cancel for the ground state, but not for other quantum states of the plasma in the sphere, which is essentially a three-dimensional quantum harmonic oscillator. In this way, the sphere behaves just as does an atom. An atom in its ground state has the attractive interaction found by Casimir and Polder, equation (1), but an atom in an excited state has a quasi-oscillatory interaction potential with the plate [8].

In section 2, the vacuum mode contribution found in [5] will be reviewed. The contribution of the charge fluctuations will be computed in section 3, and the results will be discussed in section 4.

2. Contribution of the quantized electromagnetic field

In this section, we will summarize the results of [5] concerning the force due to the vacuum modes or the quantized electromagnetic field. If one has a polarizable particle with dynamic polarizability $\alpha(\omega)$ located at a distance z from a perfectly reflecting plane, the force on the particle may be computed from the Maxwell stress tensor. The vacuum modes in the presence of the plate consist of a linear combination of an incident and a reflected wave. Each of these interacts with the particle and creates a radiated dipole field. The force on the particle arises from cross terms between (1) the reflected wave and the dipole field induced by the incident wave and (2) the incident wave and the dipole field induced by the reflected wave. After integrating over all modes, one finds the net force to be

$$F_V = -\frac{1}{4\pi z^4} \int_0^\infty d\omega \alpha_1(\omega) [3 \sin 2\omega z - 6z\omega \cos 2\omega z - 6z^2\omega^2 \sin 2\omega z + 4z^3\omega^3 \cos 2\omega z]. \quad (4)$$

Here, $\alpha_1(\omega)$ is the real part of $\alpha(\omega)$. The appearance of the real part of the dynamic polarizability is a crucial feature of this result for F_V . Equation (4) is derived using a

dipole approximation which requires the sphere to be small compared to both the distance to the plane, z , and to the characteristic wavelengths associated with $\alpha_1(\omega)$.

Now consider a sphere of radius a composed of a uniform material with dielectric function $\varepsilon(\omega)$. The complex polarizability is given by

$$\alpha(\omega) = a^3 \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2}. \quad (5)$$

We will take the dielectric function to be that of the Drude model,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \quad (6)$$

where ω_p is the plasma frequency and γ is the damping parameter. From equations (5) and (6), we find that the real part of the polarizability is given by

$$\alpha_1 = a^3 \omega_p^2 \frac{\omega_p^2 - 3\omega^2}{(3\omega^2 - \omega_p^2)^2 + 9\omega^2\gamma^2}. \quad (7)$$

Note that although $\alpha(\omega)$ has poles only in the lower half- ω plane, its real part $\alpha_1(\omega)$ has poles in both the upper and lower half planes. In the context of this model, the sphere is required to be small in the sense that $a \ll z$ and

$$a\omega_p \ll 1. \quad (8)$$

We can evaluate the integral in equation (4) by rotating the contour of integration to the imaginary ω -axis. However, in this process we will also acquire a contribution from the residue of the pole of $\alpha_1(\omega)$ at $\omega = \Omega + \frac{1}{2}i\gamma$, where

$$\Omega = \frac{1}{6} \sqrt{12\omega_p^2 - 9\gamma^2}. \quad (9)$$

Here, we are primarily interested in the limit of small damping, so we will take the limit $\gamma \rightarrow 0$ after the integrals are evaluated. In this limit,

$$\Omega = \frac{\omega_p}{\sqrt{3}}. \quad (10)$$

The force can be written as

$$F_V = J_V + P_V, \quad (11)$$

where J_V is the net contribution from integrals along the imaginary axis and P_V is that from the pole. The explicit forms of these two contributions for $\gamma = 0$ are

$$J_V = -\frac{a^3 \omega_p^2}{4\pi z^4} \int_0^\infty d\xi \frac{(3\xi^2 + \omega_p^2)(4z^3 \xi^3 + 6z^2 \xi^2 + 6z\xi + 3)}{(3\xi^2 + \omega_p^2)^2} e^{-2z\xi}, \quad (12)$$

and

$$P_V = -\frac{a^3 \omega_p^2}{24\Omega z^4} [2\Omega z (2\Omega^2 z^2 - 3) \sin(2\Omega z) + 3(2\Omega^2 z^2 - 1) \cos(2\Omega z)]. \quad (13)$$

Here, the positive direction is taken to be away from the plate, so a positive value for the force indicates repulsion.

3. Contribution of the quantum charge fluctuations

In this section, we will calculate the average force on the sphere due to quantum fluctuations of the charge density in the sphere. Here, we consider only the case where $\gamma = 0$, so the plasma oscillations are undamped and have a resonant frequency at the pole of $\epsilon(\omega)$, that is at $\omega = \Omega$ where Ω is given by equation (10). Classically, the plasma forms a three-dimensional harmonic oscillator with a frequency Ω . When the electron gas is described quantum mechanically, the plasma becomes a three-dimensional quantum harmonic oscillator.

First, let us consider a classical oscillating dipole in the presence of a perfectly reflecting plate located at $z = 0$. Let the instantaneous dipole moment be given by

$$\mathbf{p}(t) = \mathbf{p}_0 \cos \Omega t. \quad (14)$$

The electromagnetic field radiated by this dipole is reflected by the plate and returns to exert a reaction force on the dipole. Let $\mathbf{E}(t)$ and $\mathbf{B}(t)$ be the reflected electric and magnetic fields at the location of the dipole. The resulting force on the dipole is given by

$$\mathbf{F}_P = p^j \partial_j \mathbf{E} + \dot{\mathbf{p}} \times \mathbf{B}. \quad (15)$$

(This form can be derived directly from the Lorentz force law and differs slightly from the expression for the force used in [5], which was derived from the Maxwell stress tensor. However, one can show that both expressions lead to the same time-averaged force.)

We are interested in the component of time-averaged force which is normal to the plate. A lengthy but straightforward calculation leads to the result

$$F_P = \frac{p_{0x}^2 + p_{0y}^2}{32z^4} [(8\Omega^2 z^2 - 3) \cos(2\Omega z) + 2\Omega z (4\Omega^2 z^2 - 3) \sin(2\Omega z)] \\ + \frac{p_{0z}^2}{16z^4} [(4\Omega^2 z^2 - 3) \cos(2\Omega z) - 6\Omega z \sin(2\Omega z)]. \quad (16)$$

Now we need to briefly summarize the quantum mechanical description of the plasma in the sphere. Suppose that there are N free electrons in the sphere and that each electron is displaced by \mathbf{u} from its equilibrium position during the plasma oscillations. The Lagrangian for the plasma may be written as

$$\mathcal{L} = \frac{1}{2} Nm (\dot{\mathbf{u}}^2 - \Omega^2 \mathbf{u}^2), \quad (17)$$

where m is the electron mass. Thus, the system is equivalent to a single particle with mass Nm in a three-dimensional harmonic potential with resonant frequency ω . The dipole moment of the system can be expressed as

$$\mathbf{p} = Ne\mathbf{u}, \quad (18)$$

where e is the electron charge. When the system is quantized, the Cartesian components of \mathbf{u} can be expressed as

$$u_j = \frac{1}{\sqrt{2Nm\Omega}} (a_j + a_j^\dagger), \quad (19)$$

where a_j and a_j^\dagger are the annihilation and creation operators, respectively, for quanta in mode j .

We can make the link between the previous description of a classical oscillating dipole and a dipole undergoing quantum fluctuations by identifying $\langle p_j^2 \rangle$, the quantum expectation value of p_j^2 , with the time average of the corresponding squared classical field. Thus, we can set

$$p_{0j}^2 = 2\langle p_j^2 \rangle. \quad (20)$$

If the oscillator is in its ground state, we have

$$\langle p_x^2 \rangle_0 = \langle p_y^2 \rangle_0 = \langle p_z^2 \rangle_0 = \frac{1}{2NM\Omega} = \frac{a^3 \omega_p^2}{6\Omega}, \quad (21)$$

where we used the fact that

$$\omega_p^2 = \frac{4\pi N e^2}{Vm} = \frac{3N e^2}{a^3 m}, \quad (22)$$

where V is the volume of the sphere. Thus in the ground state, we find that $F_P = -P_V$. The force due to the averaged charge fluctuations exactly cancels the quasi-oscillatory term coming from the vacuum modes. The net force is then

$$F = J_V + P_V + F_P = J_V, \quad (23)$$

which is always attractive.

The situation is quite different if the quantum oscillator is not in its ground state. In this case, we can write the net force as

$$\begin{aligned} F = J_V + \frac{1}{16z^4} (\langle p_x^2 \rangle - \langle p_x^2 \rangle_0 + \langle p_y^2 \rangle - \langle p_y^2 \rangle_0) [(8\Omega^2 z^2 - 3) \cos(2\Omega z) \\ + 2\Omega z (4\Omega^2 z^2 - 3) \sin(2\Omega z)] + \frac{1}{8z^4} (\langle p_z^2 \rangle - \langle p_z^2 \rangle_0) \\ \times [(4\Omega^2 z^2 - 3) \cos(2\Omega z) - 6\Omega z \sin(2\Omega z)]. \end{aligned} \quad (24)$$

Consider, for example, the case where the state of the oscillator contains one quantum in a particular mode. For that mode, we will have $\langle p_j^2 \rangle = 2\langle p_j^2 \rangle_0$, and F will have a quasi-oscillatory part which is just minus the vacuum contribution. Note that equation (24) is equivalent to Barton's equation (4.1) [8].

A case of special interest is when the oscillator is in a coherent state of each mode. In this case, we have

$$a_j |\psi\rangle = \beta_j |\psi\rangle, \quad (25)$$

where $|\psi\rangle$ is the quantum state of the oscillator and β_j are three complex numbers. In this case, the net force becomes

$$\begin{aligned} F = J_V + \frac{2\sqrt{3}a^3 \omega_p}{48z^4} \{[(\text{Re } \beta_x)^2 + (\text{Re } \beta_y)^2] [(8\Omega^2 z^2 - 3) \cos(2\Omega z) \\ + 2\Omega z (4\Omega^2 z^2 - 3) \sin(2\Omega z)] + 2(\text{Re } \beta_z)^2 \\ \times [(4\Omega^2 z^2 - 3) \cos(2\Omega z) - 6\Omega z \sin(2\Omega z)]\}. \end{aligned} \quad (26)$$

Thus if the quantum state of the plasma is a coherent state with $|\beta_j|$ of order 1, then there is a quasi-oscillatory force on the sphere which is of the same order of magnitude as the vacuum contribution, F_V .

4. Discussion and conclusions

We have seen that plasmonic contributions, due to quantum charge fluctuations, give a significant contribution to the force on the sphere. When these modes are in their ground state, they exactly cancel the quasi-oscillatory terms P_V arising from the vacuum modes. In this case, only the vacuum mode term J_V which can be written as an integral over imaginary frequency remains. Thus, if one had ignored both the plasmon mode contribution and the residues of the poles of $\alpha_1(\omega)$, one would have inadvertently obtained the correct answer. When the plasmon modes are not in the ground state, the net force is quasi-oscillatory.

In the context of atomic systems, there is an extensive literature on the relative effects of vacuum modes and quantum charge fluctuations, often called radiation reaction or self-reaction. See, for example, [9, 10]. However, many of the treatments for atomic systems do not make it clear whether the separation between vacuum and self-reaction effects is gauge invariant. In the present model, it is obvious that the two effects are separately gauge invariant, as all of the calculations involve field strengths rather than potentials. When calculations are performed by doing second-order perturbation theory on a Hamiltonian, as in [1, 8], there is no need to consider the effects separately, and the cancellation is automatic.

In the case of the Casimir effect between planar geometries, the role of plasmonic contributions at short distances has long been recognized [11, 12]. More recently, Intravaia and Lambrecht [13] have argued that they are significant at all separations.

It seems to be desirable to understand better the physical reason for the cancellation between vacuum and plasmonic effects. It is clear that one way to upset the cancellation is to excite the plasmon modes. However, it is less clear if this is the only way in more general geometries.

Acknowledgments

I would like to thank Gabriel Barton for valuable criticism and discussions. This work was supported in part by the National Science Foundation under grant PHY-0244898.

References

- [1] Casimir H B G and Polder D 1948 *Phys. Rev.* **73** 360
- [2] Ford L H 1988 *Phys. Rev. D* **38** 528
- [3] Hacyan S, Jáuregui R, Soto F and Villarreal C 1990 *J. Phys. A: Math. Gen.* **23** 2401
- [4] Ford L H 1993 *Phys. Rev. A* **48** 2962
- [5] Ford L H 1998 *Phys. Rev. A* **58** 4279
- [6] Sopova V and Ford L H 2004 *Phys. Rev. A* **70** 062109
- [7] Barton G 2005 private communication
- [8] Barton G 1974 *J. Phys. B: At. Mol. Phys.* **7** 2134
- [9] Dalibard J, Dupontroc J and Cohen-Tannoudji C 1982 *J. Phys.* **43** 1617
- [10] Meschede D, Jhe W and Hinds E A 1990 *Phys. Rev. A* **41** 1587
- [11] Van Kampen N G, Nijboer B R A and Schram K 1968 *Phys. Lett. A* **26** 307
- [12] Barton G 1979 *Rep. Prog. Phys.* **42** 963
- [13] Intravaia F and Lambrecht A 2005 *Phys. Rev. Lett.* **94** 110404